

# Laplace Transform Time Integration Scheme in a Baroclinic Model.

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The Semi-Implicit (SI) time integration scheme is widely used in numerical weather prediction models, and is well regarded as being stable and accurate, though it has known flaws, such as the slowing down of important wave components. The Laplace Transform (LT) integration scheme can be shown to be more accurate in simple analytic analysis and when implemented in numerical models. Unlike the SI scheme, the LT scheme accurately treats important wave components and filters out high frequency components, maintaining stability and accuracy compared to the Semi-Implicit method.

Previous work has been restricted to barotropic Shallow Water (SW) models. The original LT scheme used a numerical form of the modified inverse Laplace Transform and was implemented in two SW models, Eulerian and semi-Lagrangian (Clancy & Lynch, 2011a/b). An analytic form of the modified LT inverse was later implemented in an Eulerian SW model (Lynch & Clancy 2016).

Following the scheme in Lynch & Clancy 2015, the LT scheme has now been implemented in a Hydrostatic Primitive Equation model, PEAK (Ehrendorfer 2012). This requires a transformation of the spectral equations to vertical normal modes. A number of test cases from the literature have been run at a spectral resolution of T85 to compare the LT scheme with the SI scheme, with a small time step SI run used as a reference.

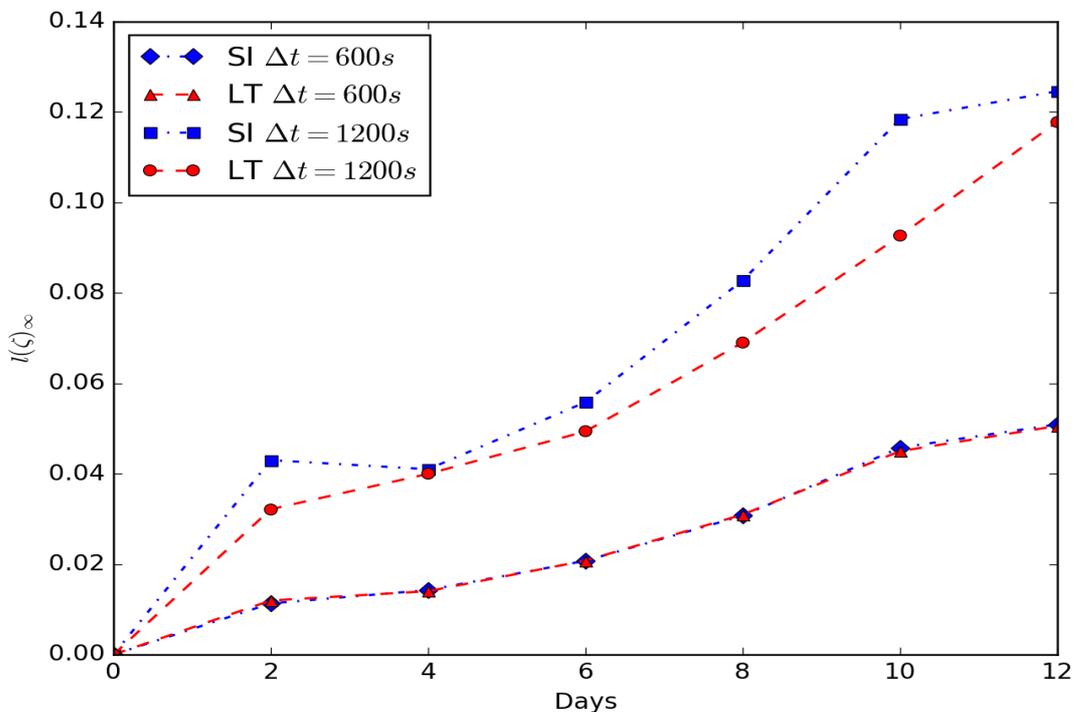


Figure 1 : Normalised  $L_{\infty}$  error for relative vorticity at the lowest model level, over 12 days for the Polvani et. al. 2004 case.

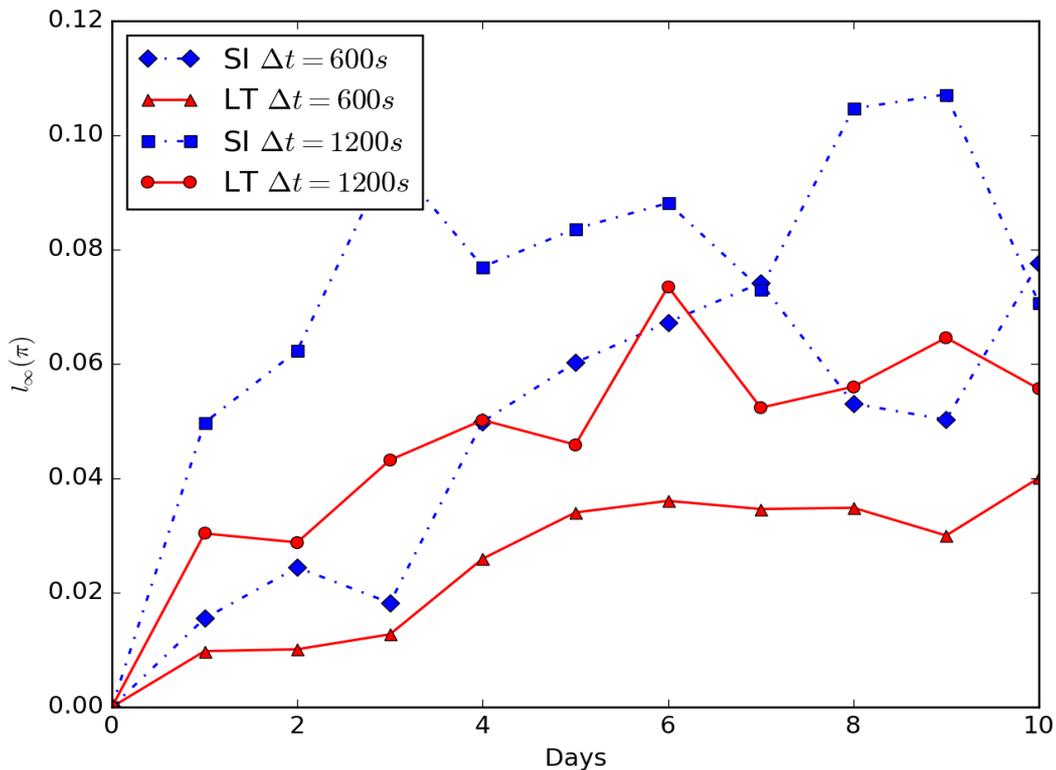


Figure 2 : Normalised  $L_{inf}$  error for log of surface pressure over 10 days for the Rossby-Haurwitz wave initial conditions from *Jablonowski et. al 2008*.

Figures 1 and 2 plot the normalised max error score ( $L_{inf}$ ) for two test cases and show the LT scheme is more accurate than the standard SI scheme, with no additional computational cost. Work is currently under way on a real data case, and following this it is hoped the scheme can be implemented in an operational forecast model.

### References:

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