Change of variable applied to mass and wind fields for covariance localisation

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1) Formulation of 3DEnVar/4DEnVar based on a common localisation

Data assimilation schemes based on either 3DEnVar or 4DEnVar formulation rely on a representation of background error covariances as a Schur product between a raw ensemble covariance matrix and a localisation matrix, in order to attenuate sampling noise which dominates at long separation distances. For efficiency reasons (e.g. Desroziers et al 2014), such formulations usually employ a common horizontal localisation function (at a given vertical level) for auto-covariances of all variables and also for associated cross-covariances. Moreover, this common horizontal localisation function usually depends on separation distance only, and it is thus isotropic.

This approach thus raises specific issues with respect to the choice of variables to which a common localisation is applied, because mass, wind and humidity fields can have different error characteristics (such as typical scales and possible anisotropy), depending on the choice of corresponding variables ; e.g. either geopotential or temperature (T) can be used for the mass field, while for the wind field, either stream function (ψ) and velocity potential (χ), either zonal (u) and meridional (v) wind, or vorticity (ζ) and divergence (η) are often considered.

2) Error characteristics and proposed change of variable for mass and wind

Typical average auto-correlation functions of zonal (respectively meridional) wind components are known to be anisotropic e.g. in the extratropical mid-troposphere (where the flow is predominantly rotational), since they tend to be zonally (respectively meridionally) elongated, in addition to be associated to specific negative lobes on the North and South sides (respectively on the East and West sides) of the considered location at the origin of the auto-correlation function (e.g. Daley 1991). Associated cross-correlation functions between zonal and meridional wind components are also anisotropic for such typical rotational flows, with a zero value at the origin and a quadripole of either positive or negative values at some distance from the origin. Such features indicate that cross-covariances of u and v are not well suited for the usual localisation based on separation distance. Different but related characteristics are expected for predominantly divergent flows, and it is only in the case of independent rotational and divergent components with equal amplitudes that zonal and meridional wind components are expected to be isotropic. Typical anisotropic cross-covariances are also classical between e.g. zonal wind and the mass field, due to geostrophic-like effects. These features suggest that zonal and meridional wind components are not much adequate variables for applying the isotropic localisation which is usually employed e.g. in 4DEnVar formulations such as in the Météo-France global model ARPEGE.

For these reasons, either vorticity and divergence, or stream function and velocity potential, are often considered for covariance modelling (e.g. Derber and Bouttier 1999) and for covariance localisation. This is related to the fact that typical average auto-correlation functions of these variables are nearly isotropic for both rotational and divergent flows. However, while typical scales of wind components are relatively similar to those of temperature and specific humidity, stream function and velocity potential are of much larger scale than T, while vorticity and divergence are of much smaller scale than T. These scale differences are directly related to the fact that vorticity and divergence (respectively stream function and velocity potential) are spatial derivatives (respectively spatial integrals) of zonal and meridional wind components.

Therefore, it would be desirable to consider wind variables which are nearly isotropic as stream function and velocity potential (and as vorticity and divergence), but which have similar spatial scales as temperature and wind components. Such wind variables can be easily constructed in spectral space for instance, after noticing that e.g. stream function and vorticity are simply related by a Laplacian operator (Δ), whose spectral coefficients Δ_n are directly proportional to the square of the total wave number n (or to $n(n+1)$ more precisely). This can be expressed as follows in terms of spectral coefficients $\zeta_{n,m}$ and $\psi_{n,m}$ (where m is the zonal wave number) :

$$\zeta_{n,m} = \Delta_n \, \psi_{n,m}$$

This Laplacian operator implies that the power spectrum of vorticity is related to the power spectrum of stream function roughly multiplied by n^4, which strongly emphasizes the contribution of large wave numbers to the power spectrum of vorticity. This suggests that isotropic wind variables, with scales intermediate between those of (ψ,χ) and (ζ,η) respectively, may be constructed by applying the square root of the Laplacian operator to stream function and velocity potential. These transformed variables ψ' and χ' may be called "scaled stream function" and "scaled velocity potential" respectively, with their spectral coefficients defined by :

$$(\psi')_{n,m} = (\sqrt{\Delta})_n \, \psi_{n,m} \qquad \text{and} \qquad (\chi')_{n,m} = (\sqrt{\Delta})_n \, \chi_{n,m}$$

This transformation preserves isotropy (because it only depends on n), and power spectra of ψ' and χ' are those of ψ and χ roughly multiplied by n^2. This is thus expected to provide nearly isotropic variables ψ' and χ', whose scales are similar to those of zonal and meridional wind components.

A similar issue of spatial scale is raised when e.g. the logarithm of surface pressure ($ln(P_s)$) is considered in addition to temperature at different vertical levels (e.g. Derber and Bouttier 1999). This logarithm of surface pressure tends to have much larger spatial scales than temperature, so that a transform may also be applied to obtain a variable with similar spatial scales as the other fields. Since surface pressure is strongly related to stream function in nearly geostrophic flows, it can be considered to apply the same square root of the Laplacian operator to $ln(P_s)$ as for ψ,χ in order to define a similarly scaled variable : $([ln(P_s)]')_{n,m} = (\sqrt{\Delta})_n \, (ln(P_s))_{n,m}$.

3) Diagnosis of horizontal localisation scales for scaled variables, and preliminary 4DEnVar experiments

In order to evaluate the adequacy of these scaled variables for applying a common horizontal localisation (at a given vertical level), some localisation length-scales have been diagnosed for a few variables at different vertical levels for the ARPEGE 4DEnVar system (Desroziers et al 2014), using a 200-member ensemble corresponding to random draws from the operational ARPEGE background error covariance matrix. Horizontal localisation length-scales have been here diagnosed using optimality-based localisation diagnostics (Ménétrier et al 2015).

The corresponding vertical profiles of length-scales (Figure 1) indicate that scaled stream function ψ' and scaled velocity potential χ' have similar localisation length-scales as temperature and humidity. A similar result is obtained for the scaled logarithm of surface pressure $[ln(P_s)]'$, whose localisation length-scale is close to those of low-level temperature, whereas the localisation length of $ln(P_s)$ is about 5 times larger than for temperature.

Preliminary single-observation assimilation experiments also indicate that e.g. the vertical coupling between low-level temperature and surface pressure is much better preserved when using such scaled variables, due to a more consistent treatment of horizontal localisation scales for surface pressure and temperature. Multivariate relationships such as geostrophy are also expected to be better represented when using these variables, due to underlying nearly isotropic cross-covariances which are more adequately localised. This is supported in the experimental ARPEGE 4DEnVar system, by reduced spinup effects when using scaled variables ψ', χ' and $[ln(P_s)]'$ instead of *u, v* and $ln(P_s)$.

These scaled variables have thus been adopted in current experimentations of the ARPEGE 4DEnVar system which is under development at Météo-France.

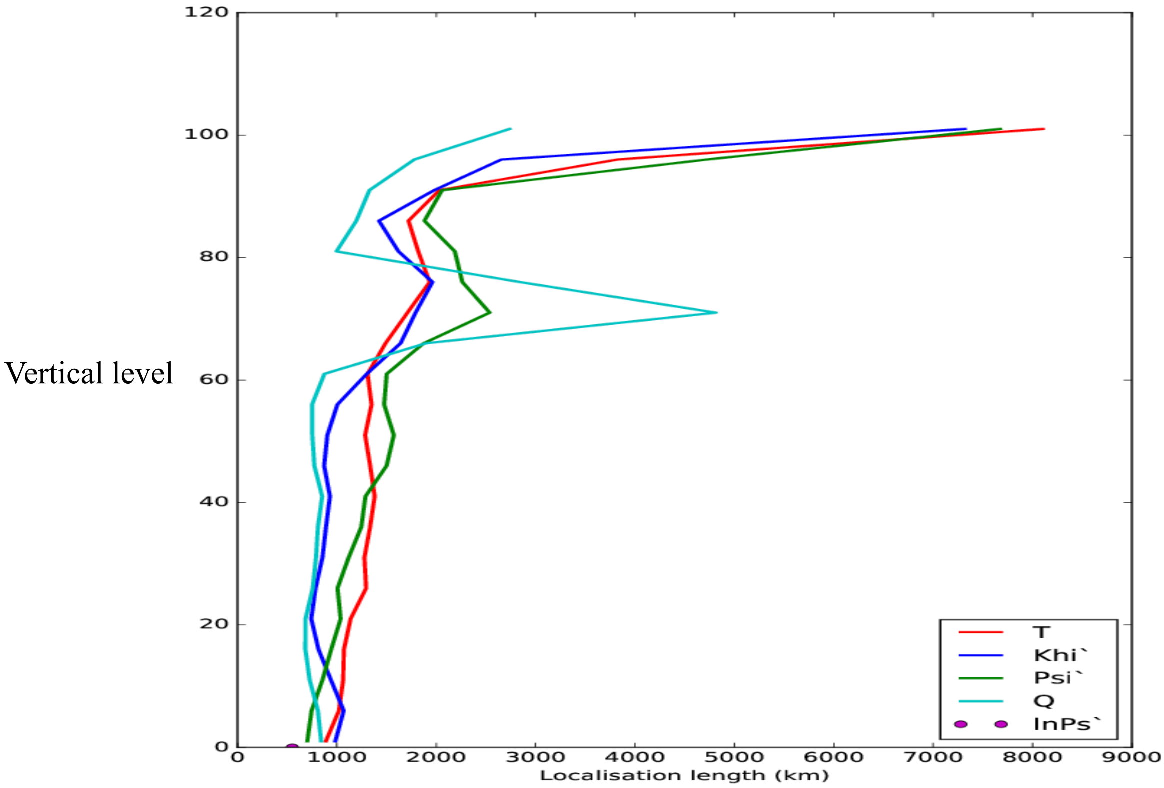

Figure 1: Vertical profile of horizontal localisation length-scales (in m) diagnosed for temperature (T), scaled stream function (Psi'), scaled velocity potential (Khi'), humidity (Q) and scaled logarithm of surface pressure (lnPs'). An average profile is used in 4DEnVar.

References

Daley, R., 1991 : Atmospheric data analysis. Cambridge University Press, Cambridge, UK, 460 pp.

Derber, J. and F. Bouttier 1999 : A reformulation of the background error covariance at ECMWF. Tellus, 51A, 195-221.

Desroziers, G., Camino, J.-T. and Berre, L. (2014), 4DEnVar: link with 4D state formulation of variational assimilation and different possible implementations. Q.J.R. Meteorol. Soc., 140: 2097–2110.

Ménétrier, B., T. Montmerle, Y. Michel, and L. Berre, 2015: Linear Filtering of Sample Covariances for Ensemble-Based Data Assimilation. Part I: Optimality Criteria and Application to Variance Filtering and Covariance Localization. *Mon. Wea. Rev.*, **143**, 1622–1643.