

# Development of a nonhydrostatic global spectral atmospheric model using double Fourier series

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## 1. Introduction

The Global Spectral Atmospheric Model (GSAM) of the Japan Meteorological Agency (JMA) is a hydrostatic spectral model using spherical harmonics. In GSAM, a two-time-level semi-implicit scheme and a vertically conservative semi-Lagrangian scheme (Yoshimura and Matsumura 2005; Yukimoto et al. 2011) are used to allow a longer timestep, and the reduced grid (Miyamoto 2006, 2009) is used to save computational cost. We have developed a nonhydrostatic version of GSAM for higher resolutions. We have also developed an option of using double Fourier series instead of spherical harmonics as spectral basis functions for efficiency.

## 2. Development of nonhydrostatic spectral model

We have developed a nonhydrostatic dynamical core for GSAM. We use the following prognostic equations:

$$\pi \equiv A(\eta) + B(\eta)\pi_s \quad (1)$$

$$P \equiv \frac{p - \pi}{\pi} \quad (2)$$

$$\frac{d(\mathbf{v} + 2\mathbf{\Omega} \times \mathbf{r})}{dt} = -RT \left( \frac{1}{\pi} \nabla \pi + \frac{1}{1+P} \nabla P \right) - \nabla \Phi - \nabla \Phi \frac{1}{\partial \pi / \partial \eta} \frac{\partial(\pi P)}{\partial \eta} \quad (3)$$

$$\frac{dw}{dt} = g \frac{1}{\partial \pi / \partial \eta} \frac{\partial(\pi P)}{\partial \eta} \quad (4)$$

$$\frac{dT}{dt} = -\frac{RT}{c_v} D_3 \quad (5)$$

$$\frac{\partial}{\partial t} \left( \frac{\partial \pi}{\partial \eta} \right) + \nabla \cdot \left( \mathbf{v} \frac{\partial \pi}{\partial \eta} \right) + \frac{\partial}{\partial \eta} \left( \dot{\eta} \frac{\partial \pi}{\partial \eta} \right) = 0 \quad (6)$$

$$\frac{1}{1+P} \frac{dP}{dt} = -\frac{1}{\pi} \frac{d\pi}{dt} - \frac{c_p}{c_v} D_3 \quad (7)$$

$$D_3 \equiv \nabla \cdot \mathbf{v} + \frac{\pi(1+P)}{RT} \nabla \Phi \cdot \left( \frac{1}{\partial \pi / \partial \eta} \frac{\partial \mathbf{v}}{\partial \eta} \right) - \frac{\pi(1+P)}{RT} \frac{g}{\partial \pi / \partial \eta} \frac{\partial w}{\partial \eta} \quad (8)$$

$$\frac{\partial \Phi}{\partial \eta} = -\frac{RT}{1+P} \frac{1}{\pi} \frac{\partial \pi}{\partial \eta} \quad (9)$$

where,  $p$  is pressure,  $\pi$  is hydrostatic pressure,  $\pi_s$  is surface hydrostatic pressure,  $\eta$  is a hybrid vertical coordinate of  $\sigma (= \pi / \pi_s)$  and  $p$ ,  $t$  is time,  $\mathbf{v}$  is the horizontal wind vector,  $w$  is vertical wind,  $T$  is temperature,  $D_3$  is 3-dimensional divergence of the wind,  $\Phi$  is geopotential height,  $\mathbf{\Omega}$  is the earth's rotation,  $\mathbf{r}$  is the radial position vector,  $R$  is gas constant,  $c_p$  is specific heat capacity at constant pressure, and  $c_v$  is specific heat capacity at constant volume. These equations are substantially the same as those of the ALADIN-NH nonhydrostatic limited-area spectral model (Bubnová et al. 1995; Bénard et al. 2010) and those of the nonhydrostatic version of IFS (Wedi and Smolarkiewicz 2009). But there are some differences in the way of integration. In Bénard et al. (2010), an iterative centered implicit (ICI) scheme, in which whole terms are treated implicitly through iteration, is used to enhance stability. In our nonhydrostatic model, a non-constant coefficient semi-implicit scheme is used, in which linear terms are treated implicitly and residual nonlinear terms are treated explicitly. The coefficients of some of the linear terms are set to be non-constant in time and space. When linearizing the underlined terms in Eqs. (3) and (8), which relate to sound waves, the present values of  $\nabla \Phi \cdot \partial \eta / \partial \pi$  and  $\pi / RT \cdot \nabla \Phi \cdot \partial \eta / \partial \pi$  are used as non-constant coefficients. These values become large where the orography is steep. Using not only constant coefficients but also non-constant ones contributes to enhance stability because the approximation by the linear terms becomes better and the residual nonlinear terms become smaller. A preconditioned generalized conjugate residual (GCR) method, a fast-convergent iteration method, is used to solve simultaneous linear equations along with the non-constant coefficient semi-implicit scheme, where a constant coefficient semi-implicit calculation is used as a preconditioner. At the 15km or coarser horizontal resolutions, only one iteration is enough to converge in our test runs. The non-constant coefficient semi-implicit scheme with the preconditioned GCR method is more efficient compared with ICI, because only the two linearized terms underlined are needed to be recalculated per iteration in grid-space.

## 3. Development of double Fourier series option

We have succeeded in developing not only a hydrostatic double Fourier series model (Yoshimura and

Matsumura 2005) but also a nonhydrostatic one. In a double Fourier series model, the fast Fourier transform is used instead of the Legendre transform, which reduces computational cost in high resolutions. We use the same type of double Fourier series as in the Eulerian global shallow water model in Cheong (2000). The coefficient of the 4th-order diffusion term (i.e. biharmonic spectral filter in Cheong 2002) in the double Fourier series GSAM is as large as in the spherical harmonics GSAM and not needed to be larger. Strong spectral filters (e.g. spherical harmonics filter) are not necessary in our semi-implicit semi-Lagrangian double Fourier series model.

#### 4. Test runs

Two-day test runs of the hydrostatic GSAM and the nonhydrostatic version of GSAM are performed from the initial conditions on 24 June 2011 at the TL1279L60 (15km horizontal grid) resolution. Figure 1 shows the 2-day mean convective and large-scale condensation precipitation of the hydrostatic and the nonhydrostatic models. The difference of precipitation between the hydrostatic and nonhydrostatic models is small. But in some mountain regions, large-scale condensation precipitation is apparently larger in the hydrostatic model than in the nonhydrostatic model. Here, only the results of the models using double Fourier series are shown. The results of the spherical harmonics and the double Fourier series models are very close. In this resolution, computational time in the double Fourier series model is about 0.7~0.8 times as long as that in the spherical harmonics model.

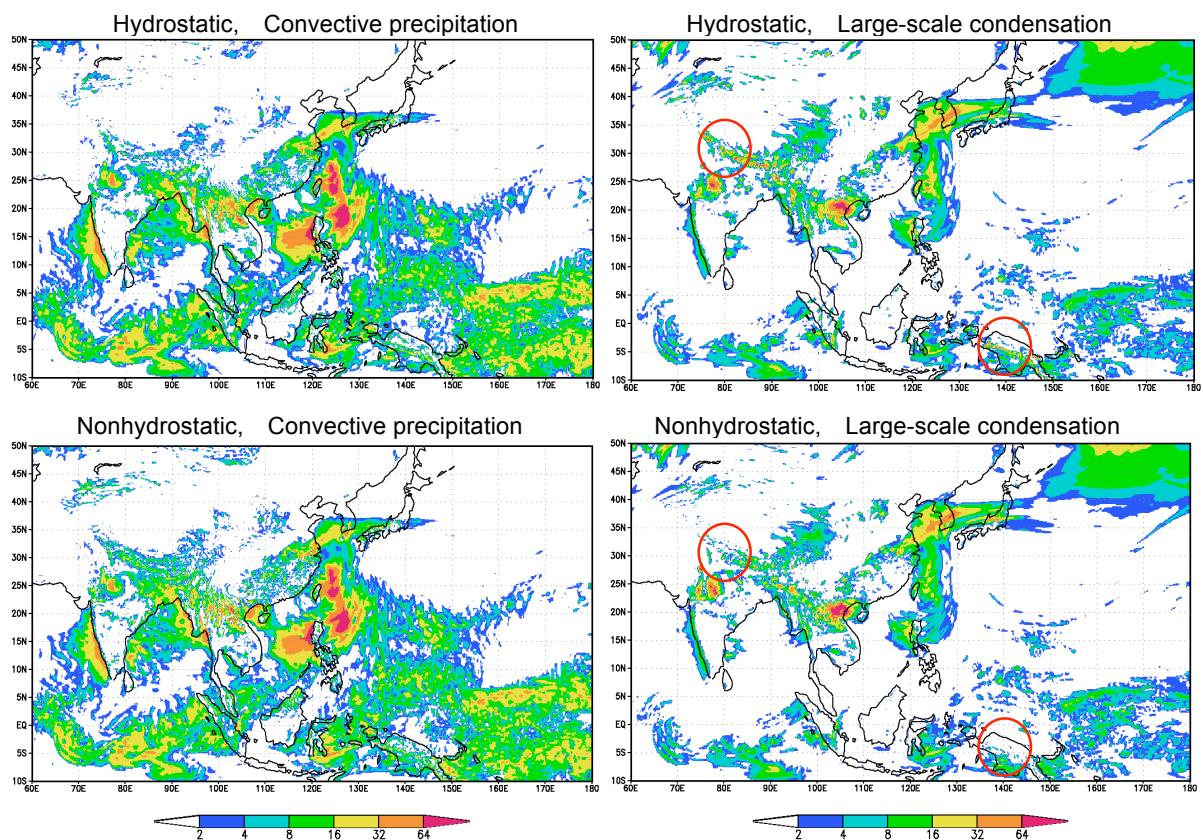


Fig. 1. 2-day mean convective and large-scale condensation precipitation of the hydrostatic and the nonhydrostatic models.

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