Turbulence length scale formulated as a function of moist Brunt-Väisälä frequency

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In the scales simulated by present NWP models all the 3D turbulent eddies are sub-grid. Still the higher vertical resolution of such models allows physically realistic parametrization of the most energetic eddies at every model level. This is usually done through the concept of turbulence length scale, becoming the only free parameter closing the turbulence scheme. It is then quite evident that the quality of a whole turbulence parametrization is strongly related to the way the length scale is formulated. In the world of NWP models, aiming to deliver realistic simulation for any specific weather type, a unique and general formulation of mixing length is always of great advantage. In this sense the formulation proposed by Bougeault and Lacarrère (1989) (hereafter BL89) seems to deliver a very attractive solution for mesoscale models by offering physically sound results regardless of any specific atmospheric stratification. In their approach the length scale L is related to the two characteristic lengths l_{up} and l_{down} obtained as a distance that a parcel originating at a given level z with an initial kinetic energy equal to the mean TKE of the layer $e(z)$ can travel upwards and downwards before being stopped by buoyancy effects. This can be mathematically expressed as:

$$\int_z^{z+l_{up}} \frac{g}{\bar{\theta}_v} \left(\bar{\theta}_v(z') - \bar{\theta}_v(z)\right) dz' \quad = e(z) \tag{1}$$

$$\int_{z+l_{down}}^{z} \frac{g}{\bar{\theta}_v} \left(\bar{\theta}_v(z) - \bar{\theta}_v(z')\right) dz' \quad = e(z), \tag{2}$$

with $\bar{\theta}_v$ and g being virtual potential temperature and gravity constant. In this respect the method offers a non-local length scale affected not only by stability at a given level but also influenced by remote stable zones. Another interesting feature of this method is that when (1) and (2) are evaluated by a second-order accuracy algorithm the length scales in uniformly stratified atmosphere converge toward the well known Deardorff (1980) length scale given as

$$l_{up} = l_{down} = \sqrt{\frac{2e}{N^2}} \quad . \tag{3}$$

This can be then interpreted that the BL89 length scale is in a way a non-local generalization of the Deardorff mixing length (Cuxart et al., 2000).

In the framework of the newly developed TOUCANS (Third Order moments scheme with Unified Condensation Accounting and N-dependent Solver) turbulence scheme the moist processes are treated through modified Richardson number Ri''. For consistency reasons it is then desirable to formulate the length scale in terms of (moist) Brunt-Väisälä frequency N_v^2 related to the Ri'' through the shear S^2: $N_v^2 = Ri''S^2$. Keeping the same discrete formulation the equations (1) and (2) can be formally rewritten to desirable form using directly N_v^2:

$$\int_z^{z+l_{up}} N_v^2(z'-z)dz' \quad = e(z) \tag{4}$$

$$\int_{z+l_{down}}^{z} N_v^2(z-z')dz' \quad = e(z). \tag{5}$$

This reformulation, on top of being consistent with the Ri'' concept, also allows to replace the costly and complicated second-order accurate evaluation of the path within the last model level that a particle is reaching by the desired Deardorff relation (3) using the N_v^2 of the actual level and the hypothesis that the remaining energy is fully spent by that displacement. This potentially makes the new method attractive also for the linearized formulation in the TL/AD models. In addition this new formulation of BL89 length scale reduces almost twice the computational cost, provided the Ri'' is already evaluated earlier in the scheme.

Part of this work was performed in the framework of the CzechGlobe project. Supports of the RC LACE Consortium as well as those of Météo-France are gratefully acknowledged.

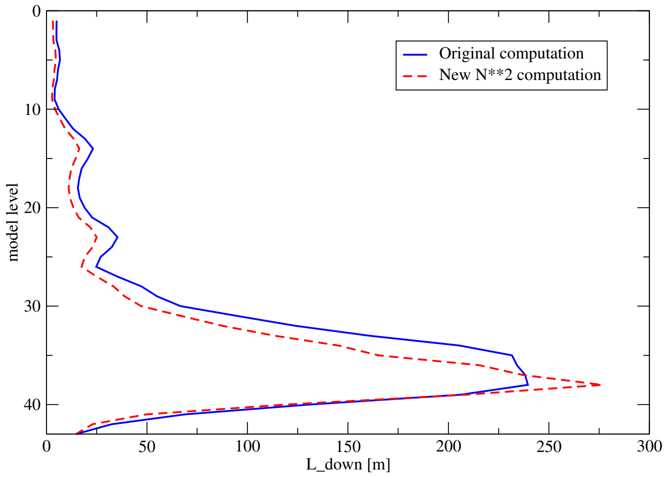

Figure 1: The downward displacement l_{down} evaluated for a randomly chosen model column by original BL89 method (blue full line) and the new method relating length scale directly to the moist Brunt-Väisälä frequency N_v^2 (dashed red curve). Although the two methods share a nearly equivalent discrete formulation the difference between the two curves originates in the way how the N_v^2 is being evaluated. In the latter case N_v^2 is directly derived from the Ri'', while the former formulation is evaluated independently of this key parameter of the TOUCANS turbulence scheme.

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