Effects of basic state update in the JMA global 4D-Var

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1. Introduction

Analyses of the atmosphere based on variational schemes (4D-Var, 3D-Var) are executed with optimization algorithms (the quasi Newton method, the conjugate gradient method) using gradient of the cost function. If the tangent linear approximation of the observation operator is valid around a basic state, the cost function becomes quadratic form. The quadratic form of the cost function is desirable for the optimization problem, and for operational data analysis systems (DASs) to be stable and keep high accuracy. Further more, in this case, since the DAS becomes a linear system, we can derive the analytical solution easily, and linear analyses of analysis and forecast results are possible. While, the DAS have to treat nonlinearity of the observation operator because observations based on remote sensing (satellite radiances, RADAR, GPS) have contributed to analysis and forecast accuracy largely, and nonlinearity of observation operators of these data is stronger than that of conventional direct observations. Furthermore, cloud and rain affected observations may bring significant improvement of analysis and forecast accuracy.

Therefore, both linearity and nonlinearity of observation operators are requested in DASs. The basic state update in incremental 4D-Var（Courtier et al. 1994）is an answer of these contradistinctive requests. The basic state update has been used in several numerical weather prediction centers, and analyses by the adjoint-based observation impact estimation method also shows effectiveness of the method（Trémolet 2008）. We describe the first test of the basic state update in the Japan Meteorological Agency (JMA) global 4D-Var DAS.

2. Formulation of basic state update

The incremental 4D-Var with the basic state update is formulated as follows;

$$J^m \cong \frac{1}{2}\left(\delta\mathbf{x}^m + \mathbf{b}^m\right)^T \mathbf{B}^{-1}\left(\delta\mathbf{x}^m + \mathbf{b}^m\right) + \frac{1}{2}\left(\mathbf{d}^m - \mathbf{H}^m\delta\mathbf{x}^m\right)^T \mathbf{R}^{-1}\left(\mathbf{d}^m - \mathbf{H}^m\delta\mathbf{x}^m\right),$$

$$\delta\mathbf{x}^m = \left(\mathbf{B}^{-1} + \mathbf{H}^{mT}\mathbf{R}^{-1}\mathbf{H}^m\right)^{-1}\mathbf{H}^{mT}\mathbf{R}^{-1}\mathbf{d}^m - \left(\mathbf{B}^{-1} + \mathbf{H}^{mT}\mathbf{R}^{-1}\mathbf{H}^m\right)^{-1}\mathbf{B}^{-1}\mathbf{b}^m,$$

where, J is the cost function, δ**x** is the analysis increment vector, **y** is the observations, **d** is the departure vector (differences between observations and guesses), **B** is the back ground error covariance matrix, **R** is the observation error covariance matrix, **H** is the tangent linear observation operator. The superscript m denotes m-th basic state update concerned quantity, and **b** is the differences between the basic field and the background field.

To recalculate departure values using a high-resolution outer model for the basic state update is computationally expensive. Here, we take more simple way using an assumption about representation errors, which is usually used in departure value calculations. First, the resolution that we can analyze is that of an inner model space. Therefore, the departure values must be differences between observations and guesses in inner model resolution. However, such observations with inner model resolution are not exist, so differences between guesses in outer model space and original observation values are used usually. This is the same as the assumption that high wave number components of guesses and observations are canceled out by the subtraction. If we separate observations into the low and the high resolution parts formally. This assumption is written, as follows;

$$\mathbf{y} = \mathbf{y}^L + \delta_y; \quad H^h\left(\mathbf{x}_b^h\right) = H^L\left(\mathbf{x}_b^L\right) + \delta_b$$
$$\mathbf{d}_b^h = \mathbf{y}^L - H^L\left(\mathbf{x}_b^L\right) + \left(\delta_y - \delta_b\right) \cong \mathbf{y}^L - H^L\left(\mathbf{x}_b^L\right) = \mathbf{d}_b^L,$$

where, the superscripts L, h, and the subscript b denote low resolution, height resolution, and background field concerned quantity, respectively. H(x) is observation operator. The last approximate equivalence means we ignore the differences between the deltas, and this corresponds to the assumption. Therefore, hypothetical low-resolution observation can be calculated as the sum of high resolution departures and low-resolution first guesses. This form is the same as the equations found in Courtier et al. (1994) and Ishikawa and Koizumi (2002, in Japanese). Using this expression, basic state updated departure values for inner resolution are given as follows;

$$\mathbf{d}_g^L = \mathbf{y}^L - H^L\left(\mathbf{x}_g^L\right) \cong \left(\mathbf{d}_b^h + H^L\left(\mathbf{x}_b^L\right)\right) - H^L\left(\mathbf{x}_g^L\right),$$

where, subscript g denotes updated basic state concerned quantity.

3. Experiment

We have implemented the basic state update scheme to the low-resolution version of the JMA global 4D-Var. An analysis and succeeding three day forecast are performed with the scheme at 00UTC 20 Jul 2009 to evaluate the effectiveness of the scheme.

The experimental results are as follows. Figure 1 shows the decreases of observation terms of the cost function for each observation dataset after the basic state update. The most large reduction rate is found in radiance data, about 12% reduction, and the next is GPS radio occultation data and relative humidity direct observations. These observations correspond to complex observation operators or water vapor concerned observations. We can also find the decreases of cost function in temperature and winds concerned direct observations. This is because these observation operators also include the NWP model as the time evolution operator. Figure 2 shows changes in forecast accuracy due to the basic state update. The figure shows forecast errors for temperature are reduced almost all area. Forecast error reduction in winds and geo-potential height is also found (Figures not shown). However, the forecast errors of humidity in the mid troposphere has the trend to increase.

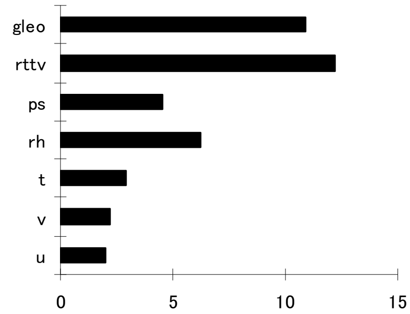

Figure 1. The decreases of observation terms of the cost function for each observation dataset due to the basic state update. The horizontal axis is the decrease rate (%), the vertical axis is observation dataset name. gleo is the GPS radio occultation data, rttv is radiances, ps is surface pressure, rh is relative humidity, u is the zonal wind, and v is the meridional wind.

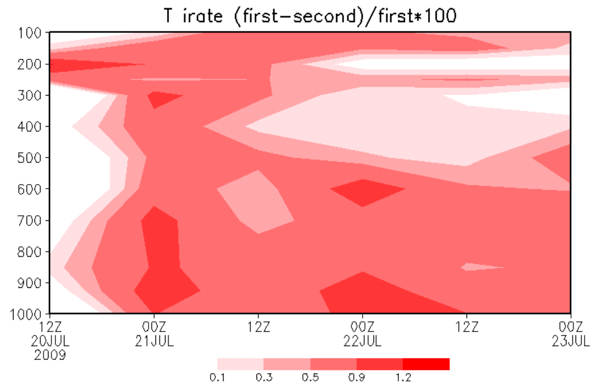

Figure 2. The improvement rate of forecast RMSE for temperature. The improvement rate is defined as (CNTL–TEST)/CNTL, where, TEST is with the basic state up date, and CNTL is without it. The horizontal axis is the forecast days (three days), the vertical axis is the pressure altitudes (hPa). The red shade denotes more than 0.1% improvement areas due to the basic state update.