Mass balance of the Antarctic ice sheet: conceptual model interpretation

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The present paper's goal is to assess the change of Antarctic ice sheet mass with the use of simple model under projected climate changes. We present conceptual model that takes into account major components of total mass balance: precipitation, melting and ice discharge. We will neglect the components of local mass balance other than precipitation and melting. Precipitation linearly depends on global surface air temperature (SAT) T.

Melting takes place in the region where surface air temperature (SAT) is positive (^oC). Local rate of melting is assumed to be proportional to SAT similar to [1]. Area S whith positive SAT depends on T, in particular, linearly. These assumptions allow to obtain the expression for melting:

$$h_m = \frac{\beta r}{v}(d + vT)^2 \tag{1}$$

Speed of ice flow across ocean-continent boundary is supposed to be proportional to third power of average thickness of grounded ice H_g (by analogy with Glen's law [2, 10]). If ice discharge is proportional to ice thickness at the ocean-continent boundary and ice sheet profile has a shape of parabola [2], then we have

$$\frac{dH_g}{dt} = (a + bT) - \frac{\beta r}{v}(d + vT)^2 - \frac{15}{8}w\left[1 - \left(\frac{H_{g0}}{H_g}\right)^2\right]^{1/2} H_g^4, \tag{2}$$

where T_1 is global average temperature at initial time t_0, τ_{cl} – time of temperature rise by 1^oC, H_{g0} – average ice thickness at which ice discharge becomes zero. First term in (2) is related with precipitation, second – melting, and third – ice discharge.

In the case of constant ice discharge and if global SAT rises linearly with time, we obtain

$$H_g(t) = H_0 + k_1(t - t_0) + k_2(t - t_0)^2 - k_3(t - t_0)^3, \tag{3}$$

where $k_1 = a + bT_1 - h_f - \frac{\beta r}{v}(d + vT_1)^2$, $k_2 = \frac{1}{2\tau_{cl}}(b - 2\beta r(d + vT_1))$, $k_3 = \frac{\beta r v}{3\tau_{cl}^2} > 0$, $k_1 = dH_g/dt|_{t=t_0}$, $k_2 = d^2H_g/dt^2|_{t=t_0}$. Ice sheet may undergo several regimes. Temporal dependence $H_g(t)$ has minimum and subsequent maximum if the following condition is satisfied

$$b > 2\beta r\left[d + \left(d^2 + (h_f - a)\frac{v}{\beta r}\right)^{1/2}\right] \tag{4}$$

Denote time of minimum as t_1, time of inflection – t_2 and time of maximum – t_3 ($t_1 < t_2 < t_3$). Fig. 1 shows straight lines corresponding to $k_1 = 0$ and $k_2 = 0$. Precipitation values of $100\,mm/yr$ and $271\,mm/yr$ were used as a samples from [3, 4, 5]. These lines constrain four zones which represent qualitatively different cases: 1) $t_0 < t_1$: $k_1 < 0$, $k_2 > 0$, 2) $t_1 < t_0 < t_2$: $k_1 > 0$, $k_2 > 0$, 3) $t_2 < t_0 < t_3$: $k_1 > 0$, $k_2 < 0$, 4) $t_0 > t_3$: $k_1 < 0$, $k_2 < 0$ (Numeration here is the same as in the Fig. 1). When $t_0 < t_1$ we have decreasing of $H_g(t)$ at $t = t_0$ and positive k_2. Situations 2) and 3) are those which have been noted in some model experiments [5, 6, 7]. According to results of abovementioned simulations, at an initial stage ice sheet grows due to dominating precipitation and shrinks thereafter according to increasing melting. Such behaviour should be attributed to the top part of the $b - r$ diagram where precipitation is significant. The last case $t_0 > t_3$ is realized if at initial time melting is enough to overcome the precipitation.

It is noteworthy that numerous ice sheet models have been developed earlier. They span from 0-dimensional [5, 8, 9] (like described one) to thermo-mechanically coupled 3-dimensional models [7, 10, 11]. Proposed model provides a clear analytical interpretation of multiple patterns in behaviour of an ice sheet.

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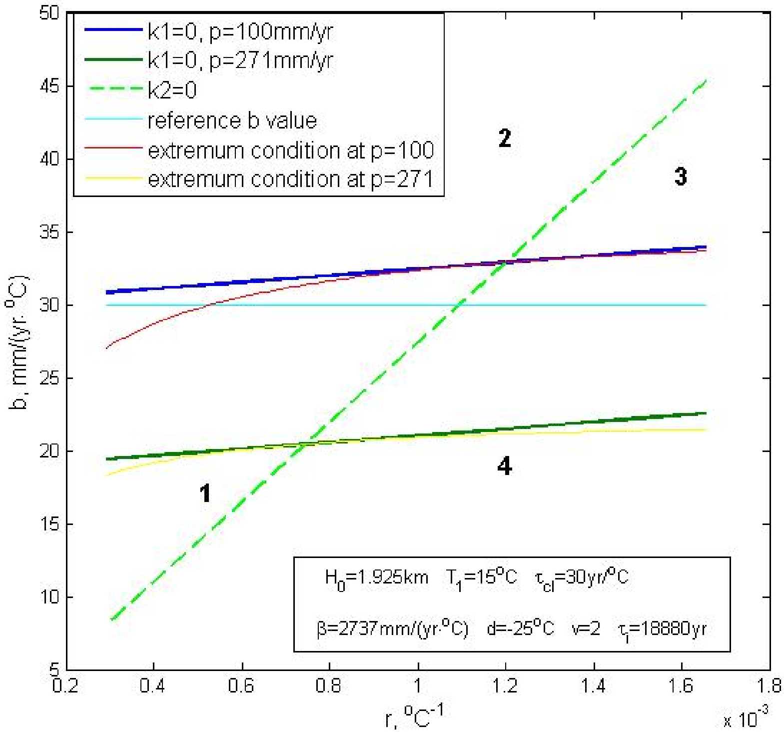

Figure 1: Ice sheet behaviour depending on parameters b and r

References

[1] M. Vizcaino, U. Mikolajewicz, M. Grögger, E. Maier-Reimer, G. Schurgers, A. Winguth (2008) Long-term ice sheet-climate interactions under anthropogenic greenhouse forcing simulated with a complex Earth System Model. Clim. Dyn.DOI 10.1007/s00382-008-0369-7.

[2] W. S. B. Paterson (1994) Physics of Glaciers. 3-rd ed., Elsevier Science Ltd., Oxford, Elsevier Science Inc., New York, Elsevier Science Japan, Tokyo, 480 p.

[3] V. N. Petrov (1975) Atmospheric gain of Antarctic ice sheet, Leningrad, Gidrometeoizdat, 152 p.(in Russian).

[4] M. Verbitsky (1992) Equilibrium ice sheet scaling in climate modeling. Clim. Dyn. 7, 105–110.

[5] I. I. Mokhov, V. K. Petukhov, I. N. Rusin (1983) Mass exchange sensitivity on the surface of Antartic ice sheet to climate change. Meteorol. Hydrol., N 11, 52–59.

[6] U. Micolajewicz, M. Vizcaino, J. Jungclaus, G. Schurgers (2007) Effect of ice-sheet interactions in anthropogenic climate change simulations. Geophys. Res. Lett., 34, L.18706.

[7] P. Huybrechts, J. Wolde (1999) The dynamic response of the Greenland and Antarctic ice sheets to multiple-century climatic warming. J. Climate, 2169–2188.

[8] E. *Källén*, C. Crafoord, M. Ghil (1979) Free oscillations in a climate model with ice-sheet dynamics. J. Atmos. Sci., 36, 2292–2303.

[9] V. G. Khodakov (1965) On dependence of total ablation of a glacier surface on air temperature. Meteorol. Hydrol., 7, 48–50.

[10] R. Greve (1997) A continuum-mechanical formulation for shallow polythermal ice sheets. Phil. Trans. R. Soc. Lond. A 355, 921–974.

[11] M. Verbitsky, B. Saltzman (1997) Modeling the Antarctic ice sheet. Annals of Glaciology, 25, 259–267.