Finite-element scheme for the vertical discretization of the global semi-lagrangian SL-AV model

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The global semi-lagrangian NWP model SL-AV [1] was developed in Hydrometeorological Research Center of Russia and Institute of Numerical Mathematics, Russian Academy of Sciences. The horizontal resolution of the model is 0.9° in longitude and 0.72° in latitude; currently there are 28 vertical levels. Model variables are vertical component of the absolute vorticity, horizontal divergence, temperature, surface pressure and specific humidity. Subgrid scale parametrizations developed in Meteo-France for ARPEGE/IFS [2] model are used.

SL-AV model validation carried out in Hydrometcentre of Russia revealed significant negative geopotential bias in the stratosphere. It is known [3] that possible reason of this bias is poor accuracy of integration of the hydrostatic equation. Untch and Hortal [3] implemented finite-element integration scheme instead of finite-difference midpoint rule and achieved significant reduction of the bias in the stratosphere.

SL-AV model previously integrated hydrostatics equation via trapezoidal rule. The analysis showed that the trapezoidal rule is more accurate than midpoint rule, however, both methods are less accurate than finite-element method with piecewise linear basis functions. Table 1 contains RMS error of the integration for $\sin 6\pi x$ function via the above methods on the 50, 100 and 150 uniform spaced levels.

Table 1

Number of points	Trapezoidal rule	Midpoint rule	Finite-element method (linear basis functions)
50	5.4575E-4	6.6815E-4	2.9030E-5
100	1.3614E-4	1.6671E-4	1.8708E-6
150	6.0480E-5	7.4069E-5	3.6577E-7

According to the table, the finite-element method is 4th order accurate while both finite-difference methods are 2nd order accurate. Furthermore, it is strictly proven that for uniform grids linear finite-element scheme is 4th order accurate [5].

Due to its high accuracy property, the finite element integration method was implemented in the SL-AV model. Hydrostatic equation and continuity equation are concerned. Two 30-day series of 120-hour forecasts for August 2005 and December 2005 using two versions of SL-AV model (with vertical integration via trapezoidal rule and via finite-element method) were calculated. The starting point for the forecasts was the analysis from [6]. The first guess is 6-hour SL-AV model forecast starting from the previous assimilation step.

Southern hemisphere averaged geopotential bias is shown in Figure 1 (finite-difference method on the left and finite-element on the right) while Southern hemisphere averaged root-mean square errors are shown in Figure 2.

One can see that integration via finite-element method reduces bias and root-mean square error in the stratosphere and upper troposphere. The results obtained from the series of forecasts of December 2005 in the Northern hemisphere are similar to the above.

Over the summer hemisphere for both (August 2005 and December 2005) forecast series bias is reduced in the upper troposphere and in the stratosphere while in the upper stratosphere positive bias is observed.

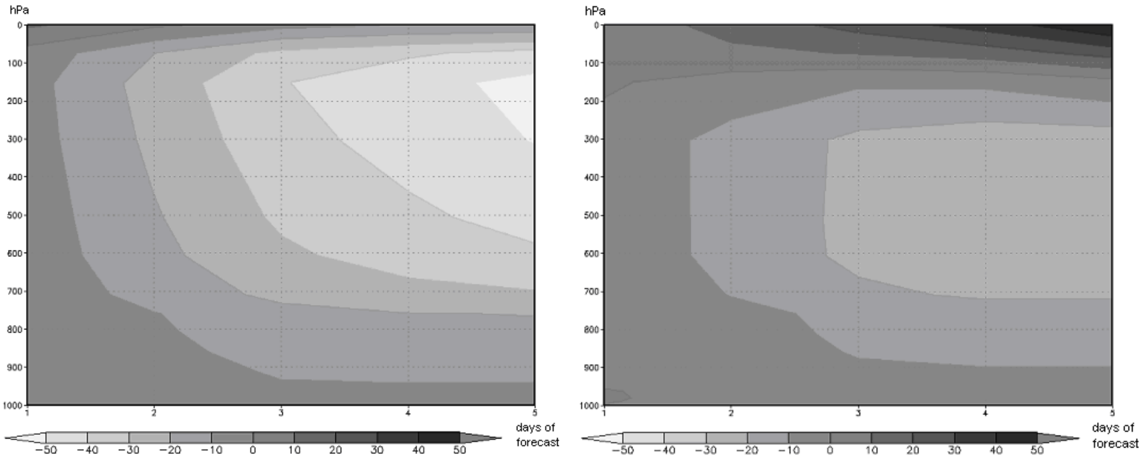

Figure 1 – Southern hemisphere averaged bias (left – finite-difference, right – finite-element)

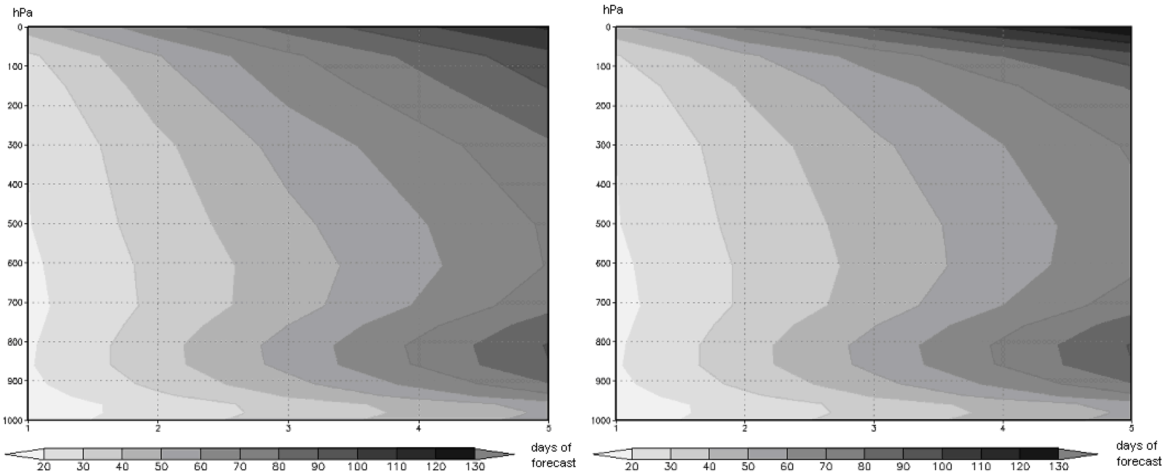

Figure 2 – Southern hemisphere averaged RMS (left – finite-difference, right – finite-element)

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