An analytical expression for the amplitude of wavenumber-one vertical velocity in the inner-core region of tropical cyclones under the influence of vertical ambient shear

Mitsuru Ueno
Typhoon Research Department, Meteorological Research Institute
e-mail: mueno@mri-jma.go.jp

It is well known that mature tropical cyclones exhibit highly axially symmetric structure in the core. On the other hand, recent observational studies (e.g., Corbosiero and Molinari 2002; Ueno 2005) have shown that convective activities tend to be enhanced on the down-shear to downshear-left side of the storm rather than evenly significant in the eyewall annulus. In the present study an analytical expression for the magnitude of wavenumber-one vertical motion asymmetries in the inner-core region of tropical cyclones under the influence of vertical ambient shear is derived in an attempt to understand the underlying mechanism for the initiation of asymmetric convection by vertical shear.

As illustrated by Jones (1995), in the situation that the vertical shear tends to tilt the vortex in the vertical, the vertical shear of the azimuthal flow tends to change in the opposite sense between the locations of maximum tangential wind and elsewhere since the vortex flow varies with radius such that the tangential wind speed increases from the center to a radius then decreases. Then consistency with thermal wind balance requires that a pair of temperature anomalies of opposite sign is generated with a positive (negative) anomaly on the upshear (downshear) side. This temperature perturbation should be achieved by vertical circulation. Based on these considerations, the vertical motion due to vertical wind shear may be quantified in the following manner.

The thermal wind balance equation for an axisymmetric typhoon-like vortex may be written in pressure coordinates as

$$\left(\boldsymbol{f}+\frac{2\boldsymbol{v}}{\boldsymbol{r}}\right)\frac{\partial \boldsymbol{v}}{\partial \boldsymbol{p}}=-\frac{\boldsymbol{R}}{\boldsymbol{p}}\left(\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{r}}\right)_{\boldsymbol{p}}, \qquad (1)$$

where $\boldsymbol{v}$ is tangential wind speed, and remaining symbols are conventional. Now consider a situation in which every portion of an initially upright vortex is being horizontally advected by the environmental wind at each level. Then the vertical shear of the azimuthal flow (i.e., $\partial \boldsymbol{v}/\partial \boldsymbol{p}$) evaluated at $\boldsymbol{p}_{\boldsymbol{M}}$ level and at a point on the downtilt side of the vortex center would change by $\partial \boldsymbol{v}/\partial \boldsymbol{r}\times \boldsymbol{S}\delta \boldsymbol{t}$ during the time period $\delta \boldsymbol{t}$, where $\boldsymbol{S}$ is the vertical shear of the environmental wind evaluated at $\boldsymbol{p}_{\boldsymbol{M}}$. Letting the corresponding temperature change at the point $\delta \boldsymbol{T}$ and assuming that Eq. (1) holds even for the tilted vortex at least locally, we may obtain

$$\left(\boldsymbol{f}+\frac{2\boldsymbol{v}}{\boldsymbol{r}}\right)\left(\frac{\partial \boldsymbol{v}}{\partial \boldsymbol{r}}\right)_{\boldsymbol{p}}\boldsymbol{S}\delta \boldsymbol{t}=-\frac{\boldsymbol{R}}{\boldsymbol{p}}\left(\frac{\partial \delta \boldsymbol{T}}{\partial \boldsymbol{r}}\right)_{\boldsymbol{p}}.$$

Assuming that the temperature tendency is totally accounted for by the temperature change due to vertical motion within the framework of dry adiabatic dynamics, and solid-body rotation to characterize the winds inside the radius of maximum wind, we can finally arrive at the following simple formula for the azimuthal wavenumber-one component of vertical velocity at the radius of maximum wind $\boldsymbol{r_m}$,

$$\omega_1 = -\frac{\boldsymbol{p}}{\boldsymbol{R}}\boldsymbol{v_m}\left(\boldsymbol{f} + \frac{2\boldsymbol{v_m}}{\boldsymbol{r_m}}\right)\boldsymbol{S} \Bigg/ \left(\frac{\kappa \boldsymbol{T}}{\boldsymbol{p}} - \frac{\partial \boldsymbol{T}}{\partial \boldsymbol{p}}\right),$$

where $\boldsymbol{v_m}$ is the maximum tangential wind at $\boldsymbol{p_M}$.

Next we attempt to validate the analytical solution obtained above by numerical model results. Figure 1 shows a time series of the magnitude of azimuthal wavenumber-one vertical motion, comparing the analytical solutions with the results from some dry model integrations. Details of the present study can be found in Ueno (2007).

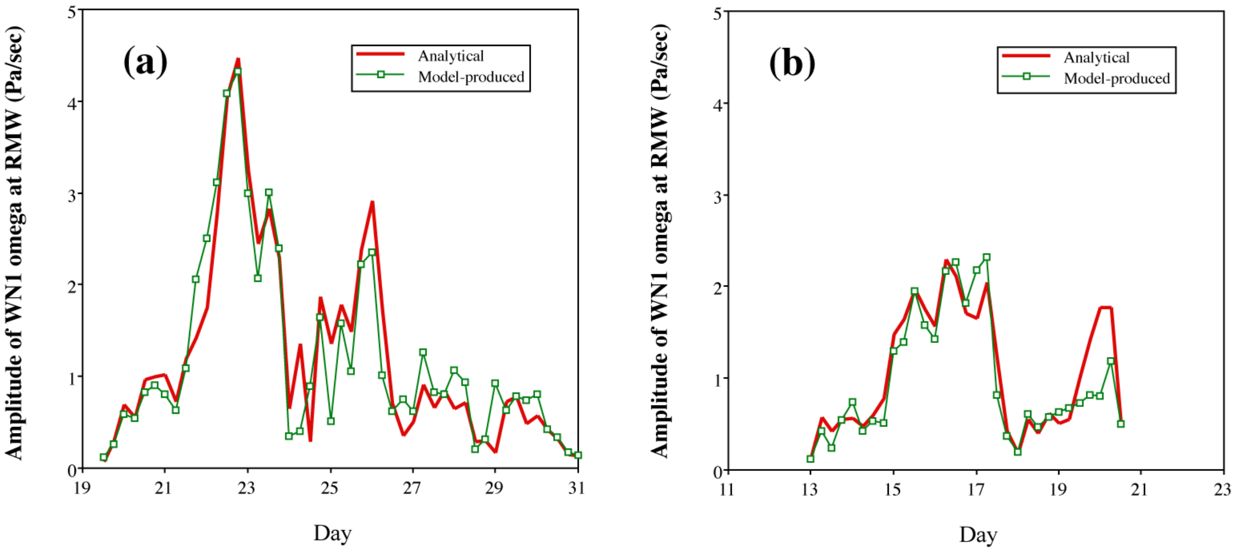

Figure 1: Comparison of "analytical" (thick red line) and model-simulated (thin green line with open squares) wavenumber-one omega (i.e., vertical p-velocity in units of Pa s^{-1}) at RMW and at 1 h of the consecutive 6-hourly dry integrations for (a) Typhoon Chaba and (b) Typhoon Tokage in 2004. The horizontal axis denotes the initial time of the integrations with numbers indicating day of the month, that is, August for (a) and October for (b), located at 00 UTC.

References

Corbosiero, K. L., and J. Molinari, 2002: The effects of vertical wind shear on the distribution of convection in tropical cyclones. ***Mon. Wea. Rev.***, **130**, 2110-2123.

Jones, S. C., 1995: The evolution of vortices in vertical shear. I: Initially barotropic vortices. ***Quart. J. Roy. Meteor. Soc.***, **121**, 821-851.

Ueno, M., 2005: Relationship between storm motion, vertical wind shear and rainfall asymmetries in typhoons. ***Kaiyo Monthly*** , **42** (Special Volume), 120-126 (in Japanese).

Ueno, M., 2007: Observational analysis and numerical evaluation of the effects of vertical wind shear on the rainfall asymmetry in the typhoon inner-core region. (***J. Meteor. Soc. Japan***, in press)