Optimized Schwarz methods with an overset grid system for the Shallow-Water Equations

Abdessamad Qaddouri[1], Lahcen Laayouni[2], Jean Côté[1,4] and Martin Gander[2,3]
1-Recherche en prévision numérique, Meteorological Service of Canada, Dorval, QC, H9P 1J3 Canada
2-Department of Mathematics and Statistics, McGill University, Montreal, Canada, QC, H3A 2K6 Canada
3-Section de Mathématiques, Université de Genève, Genève, CH-1211 Switzerland
4-Ouranos/UQÀM, 19th floor, 550 Sherbrooke West, Montreal, QC, H3A 1B9 Canada

Abstract. The overset grid system nicknamed "Yin-Yang" grid (Kageyama & Sato, 2004) is singularity free and has quasi-uniform grid spacing. It is composed of two identical latitude/longitude orthogonal grid panels that are combined to cover the sphere with partial overlap on their boundaries. The system of Shallow-Water equations (SWEs) is a hyperbolic system at the core of many models of the atmosphere. In this paper, the SWEs are solved on the Yin-Yang grid by using a semi-implicit and semi-Lagrangian discretization on a staggered mesh. The scalar elliptic equation is solved using a Schwarz-type domain decomposition method, known as the optimized Schwarz method, which gives better performance than the classical Schwarz method by using specific Robin or higher order interface conditions.

1. Introduction

The same fully implicit semi-Lagrangian method as in the GEM operational model is used to discretize the shallow-water equations as in Mahidjiba et al. (2005) except for the use of spherical trajectories: uniform Arakawa staggered C-Grid, 2-time-level iterative semi-Lagrangian method with interpolated in time advecting wind, iterative non-linear solver for the Helmholtz problem, iterative treatment of the Coriolis terms by grouping them with the non-linear terms, metric terms using the Lagrange multiplier approach of Côté (1988). This discretization is implemented independently on each quasi-uniform lat/long part grid. The trajectories are computed for each grid panel in three-dimensional Cartesian geometry with the restriction that the trajectories are confined on the surface of the sphere. The value at an upstream point is determined by the cubic Lagrange interpolation either in Yin (if this point is in Yin) or Yang grid panel. The semi-implicit treatment of the gravity terms in the SWEs gives rise to a 2D elliptic boundary value problem that must be solved at each time step. We use in this work the domain decomposition method, where the solution of the global elliptic problem is obtained by iteratively solving the corresponding two sub-problems separately on the Yin and Yang grids, and updating the values at the interfaces boundaries. The classical alternating Schwarz method consists in using each sub-problem's updated solution as boundary condition to the other one. Because the two grids do not match, the update is done with a cubic Lagrange interpolation and this corresponds to Dirichlet interface conditions. The use of specific Robin or higher order interface conditions improves the convergence of the elliptic solver.

2. Preliminary results

In the first experiment, a cosine bell is advected once around the sphere. This simulation is carried out with a resolution of 150×50 on the Yin grid and 150×50 on the Yang grid, this is equivalent to a global horizontal resolution of about 200 km. A time step of two hours is used, and it requires 144 time steps (288 hrs) to rotate the cosine hill one full revolution around Earth. Fig. 1 shows that there is no distortion in the shape of the hill at the end of the simulation. The bell structure is maintained in the Yin-Yang grid system even when the bell passes trough the overlap region. The time evolution of the normalized maximum difference is presented in Fig. 2 and as can be seen the trend and values of the norm are comparable to those in Jacob-Chien et al. (1995), and the maximum difference after 12 days is small and is about 2%.

For the next experiment we compare the convergence of the iterative elliptic solver for Dirichlet and Robin interface boundaries conditions respectively. Fig. 3 shows that the convergence is improved when Robin interface conditions are used. In this experiment the Helmholtz coefficient is equal to one (hard case) and we take the coefficient in the Robin condition also equal to one. The optimal value of this later coefficient can be found numerically and is expected to further improve the convergence.

E-Mail : abdessamad.qaddouri@ec.gc.ca

3. Conclusion

In this work we show that numerical algorithms already validated for a global latitude/longitude grid can be implemented, with minor changes, for the Yin-Yang grid system while preserving the same temporal and spatial errors. In the near future we will implement optimal Robin and second order interface conditions in order to improve the convergence of the elliptic solver, and the remaining shallow water test cases will be conducted. A parallelization strategy for this model will also be examined.

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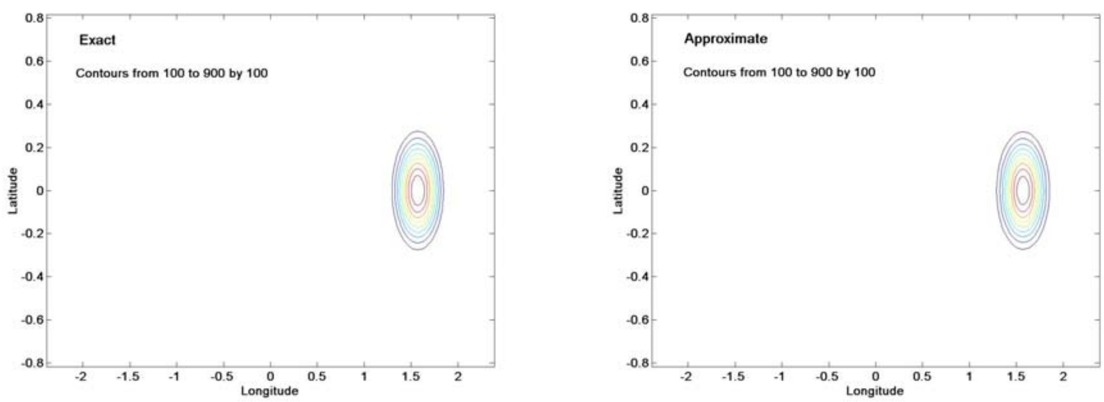

Figure 1: *Exact and approximate scalar field after one revolution around the sphere*

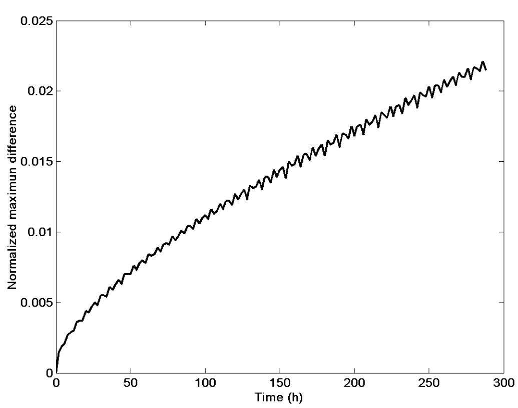

Figure 2: *Evolution of the normalized maximum difference during one revolution*

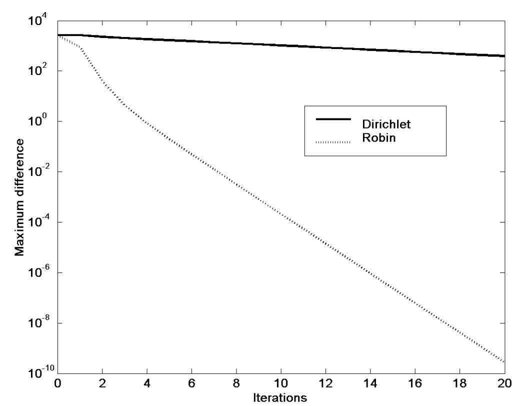

Figure 3: *Comparison of Dirichlet and Robin interface conditions on the convergence of the elliptic solver*