The reduced grid for the global semi–Lagrangian numerical weather prediction model

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The nodes of the regular latitude–longitude grid become denser as the meridians converge towards the poles. It is well known that the parameterizations of sub–grid processes in the forecast model may work incorrectly on the non–isotropic grid. The method of reduced grid constructing for a spectral model using asymptotic properties of the associated Legendre functions was proposed in [1] and later was modified in [2], [3]. The longitude step of such a grid depends on the latitude. In the present work we describe another approach suitable for semi–Lagrangian finite difference models.

The accuracy of the semi–Lagrangian scheme substantially depends on the interpolation procedure. Thus, the main goal is that to minimize the number of the grid nodes at the fixed upper limit ϵ_Φ:

$$\int_{-\pi/2}^{\pi/2} |\Phi - \Phi_0| \, d\phi_0 \Big/ \int_{-\pi/2}^{\pi/2} \Phi_0 \, d\phi_0 \leq \epsilon_\Phi, \tag{1}$$

where $\Phi(\phi_0)$ is the r. m. s. interpolation error of a given symmetric feature $f(\phi_0, \lambda, \phi)$ on the reduced grid and $\Phi_0(\phi_0)$ is calculated on the regular grid. The symmetric feature $f(\phi_0, \lambda, \phi) = \exp(-\mu d^2)$ with center at the point $(0, \phi_0)$ was found to be useful for comparison of different reduced grids. Here d is the distance on the sphere between the center $(0, \phi_0)$ and the point (λ, ϕ); $\mu = -4\ln(10^{-7})/(10.5\Delta\phi)^2$. The radius of the sphere is $a = 1$ and $\Delta\phi$ is the fixed latitude step of the grid.

Equation (1) is solved numerically for each value of ϵ_Φ and we call the grid obtained in such a way as optimal. We found that for the optimal reduced grid the quantity ϵ_Φ depends exponentially on relative reduction of the total number of nodes $n_{\rm rel}$ with respect to the regular grid.

The shallow–water test of Williamson *et al.* [4] which involves the solid–body rotation of a cosine bell around the sphere through the poles demonstrated promising perspectives of our method. The results for the number of latitudes $n_\phi = 125$ and the number of longitudes on the equator $n_\lambda = 200$ are shown in Fig. 1. The quantities Δ_i $(i = 1, 2)$ are normalized r. m. s. errors of the numerical solution calculated on the reduced grid with respect to the exact solution $(i = 1)$ and the numerical solution obtained on the regular grid $(i = 2)$. It should be noted that curves with $n_{\rm rel} = 35.4$ and $n_{\rm rel} = 29.7$ do not represent the optimal grids and the first of them is the reduced grid with constant longitude step on the sphere surface for all latitudes.

The method will be used to construct the reduced grid for the global semi–Lagrangian numerical weather prediction SL–AV model [5]. Latitudinal derivatives in this model are calculated in the space of longitudinal Fourier coefficients, so the reduced grid can be implemented.

Advantage of our method is that it allows us to take into account various details of the weather prediction model and to apply additional restrictions to the grid. See [6] for comprehensive description of the method. This work was supported by the RFBR grant 04–05–64638.

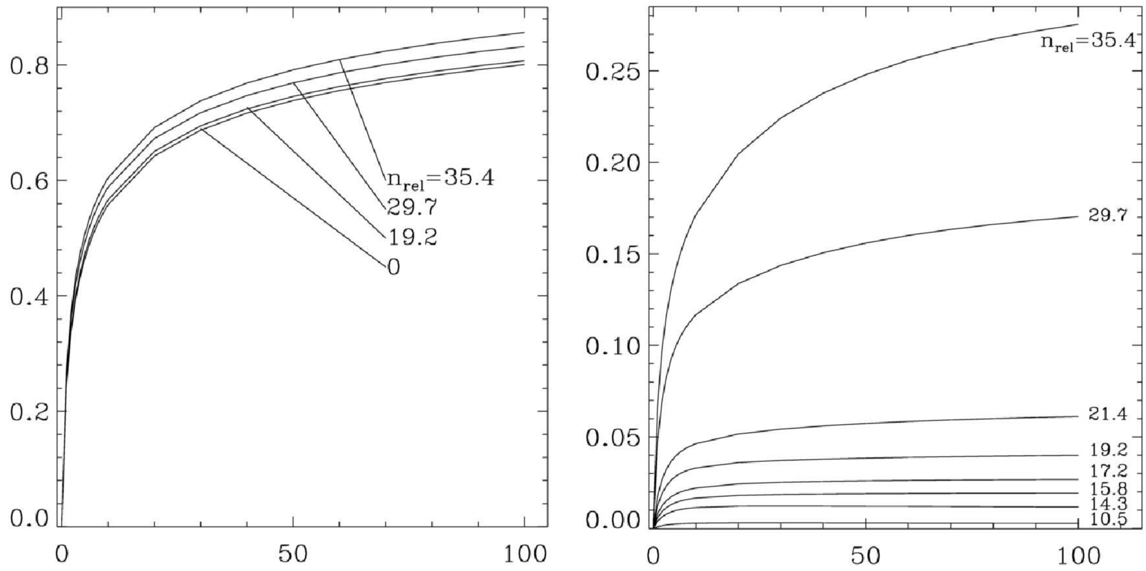

Figure 1: The quantities Δ_1 (left panel) and Δ_2 (right panel) versus the number of complete rotations (255 steps each) on the grids with different relative reduction of the total number of nodes $n_{\rm rel}$ (in per cent).

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