A Semi-Lagrangian Scheme Conservative in the Vertical Direction

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1. Introduction

The conservation of a semi-Lagrangian advection scheme is a considerable issue for a climate model. We develop a new semi-Lagrangian scheme for a climate model with a conserving property in vertical advection.

2. The semi-Lagrangian advection scheme conservative in the vertical direction

In the new semi-Lagrangian advection scheme, computation of the advection terms is split into the horizontal and vertical directions and the both terms are computed separately. The flux in the vertical direction is evaluated with a one-dimensional conservative semi-Lagrangian scheme, while the horizontal advection is calculated with a conventional non-conservative 2-dimensional semi-Lagrangian scheme. Note that we can conserve mass and water vapor when we adopt a correction method such as Gravel et al. (1994). The new advection scheme also has the advantage of computational efficiency, since it reduces the number of spatial interpolation.

The continuity equation is

$$\frac{\partial}{\partial t}\left(\frac{\partial p}{\partial \eta}\right) + \nabla \cdot \left(\mathbf{v}_H \frac{\partial p}{\partial \eta}\right) + \frac{\partial}{\partial \eta}\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right) = 0, \tag{2.1}$$

and the moisture equation is

$$\frac{\partial}{\partial t} q + \mathbf{v}_H \cdot \nabla q + \dot{\eta}\frac{\partial}{\partial \eta} q = 0, \tag{2.2}$$

where p is the pressure, q is the mixing ration of water vapor, $\mathbf{v}_H$ is the horizontal wind vector and η is the hybrid vertical coordinate of p and $\sigma (= p/p_s)$.

From (2.1) and (2.2) we obtain

$$\frac{d_H}{dt}\left(\frac{\partial p}{\partial \eta}\right) = -\left(\nabla \cdot \mathbf{v}_H\right)\left(\frac{\partial p}{\partial \eta}\right) - \frac{\partial}{\partial \eta}\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right), \tag{2.3}$$

$$\frac{d_H}{dt}\left(\frac{\partial p}{\partial \eta} q\right) = -\left(\nabla \cdot \mathbf{v}_H\right)\left(\frac{\partial p}{\partial \eta} q\right) - \frac{\partial}{\partial \eta}\left(\dot{\eta}\frac{\partial p}{\partial \eta} q\right). \tag{2.4}$$

In (2.3) and (2.4), the term on the left hand side $d_H/dt \equiv \partial_H/\partial t + \mathbf{v}_H \cdot \nabla$ is the horizontal part of the advection, the first term on the right hand side is the divergence and the second term on the right hand side is the vertical flux.

The discrete analogs of (2.3) and (2.4) are

$$\frac{d_H}{dt}\left(\Delta p_k\right) = -\left(\nabla \cdot \mathbf{v}_H\right)_k \left(\Delta p_k\right) - \left[\dot{\eta}\frac{\partial p}{\partial \eta}\right]_{k+1/2} + \left[\dot{\eta}\frac{\partial p}{\partial \eta}\right]_{k-1/2}, \tag{2.5}$$

$$\frac{d_H}{dt}\left(\Delta p_k q_k\right) = -\left(\nabla \cdot \mathbf{v}_H\right)_k \left(\Delta p_k q_k\right) - \left[\dot{\eta}\frac{\partial p}{\partial \eta} q\right]_{k+1/2} + \left[\dot{\eta}\frac{\partial p}{\partial \eta} q\right]_{k-1/2}, \tag{2.6}$$

where k is the vertical level.

We integrate (2.3) and (2.4) in time in the order as follows:

- The divergence at the departure point at time $(t-\Delta t)$
- The vertical flux at the departure point at time $(t-\Delta t)$
- The horizontal advection (with a conventional 2-dimensional semi-Lagrangian scheme)
- The vertical flux at the arrival point at time $(t+\Delta t)$
- The divergence at the arrival point at time $(t+\Delta t)$

The flux of the water vapor in the vertical direction is evaluated with a one-dimensional conservative semi-Lagrangian scheme.

The advection terms of the moisture equation should be computed consistently with that of the continuity equation. In normal semi-Lagrangian schemes, only the continuity equation is split to compute its advection terms and this may cause inconsistency between mass and other non-split variables such as water vapor. In this report we split the advection terms of the moisture equation and compute them in a similar manner to the continuity equation, ensuring the consistency on treatment of advection between the continuity and the moisture equations. The advections of the temperature and the horizontal wind components are also computed in the same way as the water vapor.

3. 3-years runs at the resolution T42L40

We performed three 3-years runs at the resolution T42L40 and compared the results. One of three was an Eulerian advection scheme, another one was a conventional semi-Lagrangian advection scheme (Matsumura 2002), and the last one was the vertically conservative semi-Lagrangian advection scheme proposed here.

In both semi-Lagrangian schemes, a time step Δt is set to 45 min, which is about twice as long as that determined by the Courant number.

Results of the 3-years runs are summarized as follows:

- With respect to the 3-years average (i.e. climate), cooling bias appears around the tropical tropopause in the conventional semi-Lagrangian scheme (Fig. 1), while it does not in the vertically conservative semi-Lagrangian scheme (Fig. 2).
- The amount of the false heat source in the atmosphere, which is calculated from the difference between the 3-years average of the total energy flux at the top and the one at the bottom, is smallest in the vertically conservative semi-Lagrangian scheme. This means that the vertically conservative semi-Lagrangian scheme is best among the three with respect to the conserving property of energy.
- In the 24-hours forecast, the difference of sea level pressure between the two semi-Lagrangian schemes is small.

These results show that the vertically conservative semi-Lagrangian scheme is better than the conventional semi-Lagrangian scheme for a long-term integration.

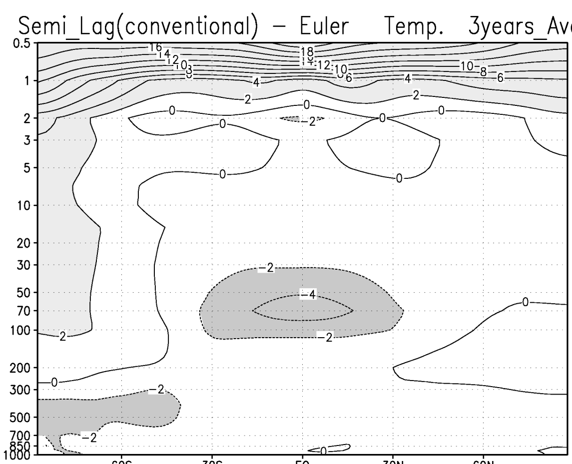

Fig. 1. The zonal mean temperature difference between the conventional semi-Lagrangian scheme and the Eulerian scheme.

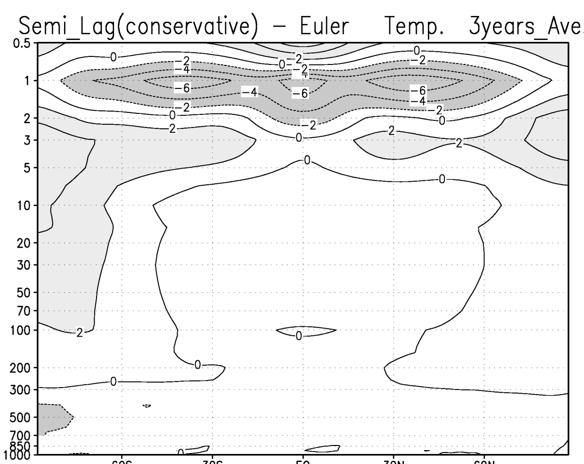

Fig. 2. The same as Fig. 1 except between vertically conservative semi-Lagrangian scheme and the Eulerian scheme.