HIGHER ORDER FINITE DIFFERENCE SCHEMES FOR ADVECTION OF NHM

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The nonhydrostatic model (NHM) of JMA, which has been developed for the very short-range prediction of rainfall, has been used mainly with a second-order finite difference scheme (Ishida 2001; Fujita *et al*. 2002). Two types of higher-order accurate schemes are implemented and tested. Since flux form equations are employed in NHM (Saito et al. 2001), the straightforward second order stencil is defined as

$$\frac{\partial U\theta}{\partial x} = \frac{\left(U\overline{\theta}\right)_{i+1/2} - \left(U\overline{\theta}\right)_{i-1/2}}{\Delta x},$$

where overbars denote horizontal interpolation. Higher-order schemes are derived referring wider range in the same direction. A fourth-order scheme is written as follows.

$$\frac{\partial U\theta}{\partial x} = \frac{9}{8}\frac{\left(U\overline{\theta}\right)_{i+1/2} - \left(U\overline{\theta}\right)_{i-1/2}}{\Delta x} - \frac{1}{8}\frac{\left(U\overline{\theta}\right)_{i+3/2} - \left(U\overline{\theta}\right)_{i-3/2}}{3\Delta x}$$

In the above equation interpolation processes should be done with high-order accuracy to guarantee the accuracy of the stencil, and the divergence term should be also computed accurately to keep conservation property in the isentropic case. Third- and fifth-order schemes require interpolation for not only θ, but also U. The third-order scheme is defined as follows.

$$\frac{\partial U\theta}{\partial x} = \frac{1}{12\Delta x}\left\{\left(U\overline{\theta}\right)_{i-3/2} - 18\left(U\overline{\theta}\right)_{i-1/2} + 8\left(\overline{U}\theta\right)_{i} + 9\left(U\overline{\theta}\right)_{i+1/2}\right\}$$

Truncation errors of the schemes above are smaller than the counterparts of the schemes in unstaggered coordinate, and subsequently the dispersive and diffusive effect of the staggered schemes is smaller.

Alternative schemes, which are used in WRF (Wicker and Skamarock 2002), are derived by considering numerical fluxes through the cell interfaces. A fourth-order scheme is defined as follows.

$$\frac{\partial U\theta}{\partial x} = \frac{F^{4\text{th}}_{i+1/2} - F^{4\text{th}}_{i-1/2}}{\Delta x} \qquad \text{where} \qquad F^{4\text{th}}_{i-1/2} = \frac{U_{i-1/2}}{12}\left\{7\left(\theta_i + \theta_{i-1}\right) - \left(\theta_{i+1} + \theta_{i-2}\right)\right\}$$

The accuracy of this scheme is guaranteed when the flow field is uniform. Schemes of different accuracy are constructed in a similar manner. These schemes will be called 'Cell Interface Difference' (CID) in this manuscript.

Figure 1 shows results of a comparison experiment of realistic simulations by NHM. The model is initialized at 00 UTC December 4, 2002, and run with 10 km horizontal resolution over the eastern part of Japan. The second-order scheme produces short-wave structures in the accumulated rainfall field, while the third-order scheme gives a smooth distribution mainly due to the numerical diffusion implied in the difference scheme. The fourth-order staggered scheme also yields a smoother field than the second-order, which suggests the higher-order scheme affects the prediction by the less dispersive characteristic. The high-order interpolation implied in the stencils may also have an effect on the fields to some extent. In the meantime, large-scale fields such as the sea level pressure field are not distinctively different. The results of the CID schemes hold similar property, however, the fourth-order CID yields not as smooth an accumulated rainfall field as the counterpart of the staggered scheme. The reason of the difference may be attributed to the difference of the actual order of accuracy between the two schemes and the effect of interpolation implied in the staggered scheme.

The computational costs of the higher-order schemes in elapse time were compared, and the results almost justified elimination of the third-order staggered scheme from the choices since it was more costly than the fourth-order staggered scheme, and as costly as the fifth-order staggered scheme. The larger cost of the third-order staggered scheme perhaps comes from the extra interpolation cost for U. The CID schemes work economically since interpolation cost is minimized in the stencil and high-order treatment for the divergence term is unnecessary.

Overall, the fourth-order staggered scheme is favored for the coming operational run of NHM, and the fourth-order CID may be an alternative.

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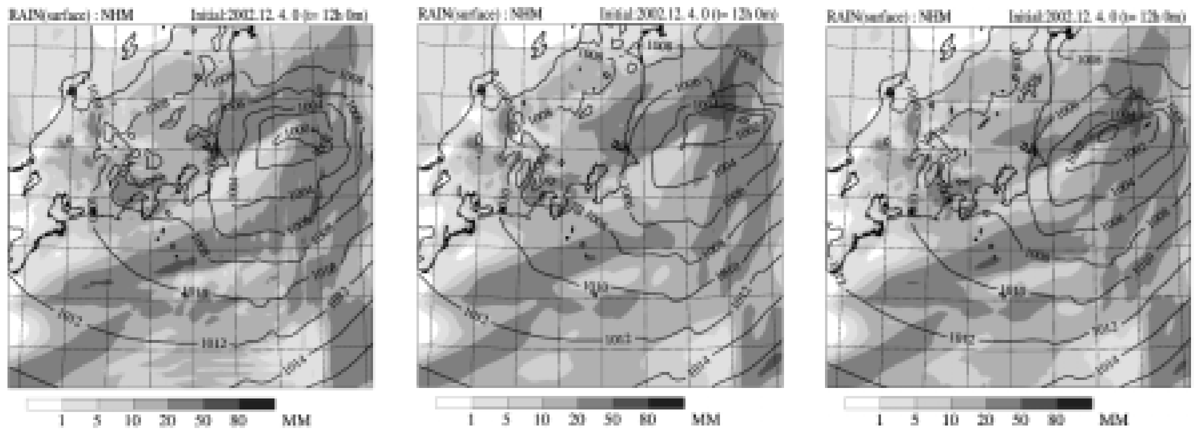
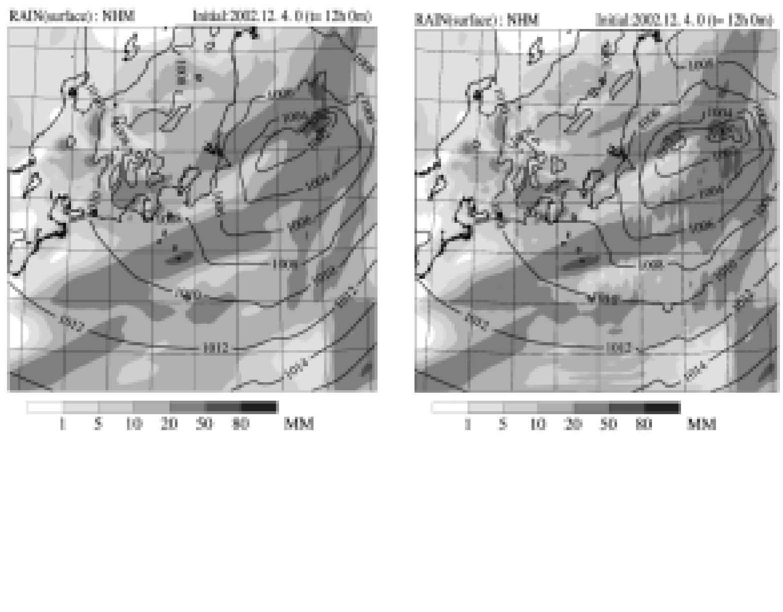

Figure 1 Sea level pressure fields and 12-hour accumulated rainfall fields of 12-hour NHM prediction initialized at 00 UTC December 4, 2002. Upper left panel shows the result of the second-order staggered scheme, upper middle the third-order, upper right the fourth-order. Lower panels show the result of third- (left) and fourth-order (right) CID schemes.